INDIAN SCHOOL AL WADI AL KABIR

Assessment - 1

Class: XII | Sub: MATHEMATICS (041) | Max Marks: 80

Date: 22.09.2024 | Set - II | Time: 3 hr

General Instructions:

1. This question paper is divided in to 5 sections- A, B, C, D and E
2. Section A comprises of 20 MCQ type questions of 1 mark each.
3. Section B comprises of 5 Very Short Answer Type Questions of 2 marks each.
4. Section C comprises of 6 Short Answer Type Questions of 3 marks each.
5. Section D comprises of 4 Long Answer Type Questions of 5 marks each.
6. Section E comprises of 3 source based / case based / passage-based questions (4 marks each) with sub parts.
7. Internal choice has been provided for certain questions
8. This question paper contains 5 pages

SECTION – A

(Each MCQ Carries 1 Mark)

1 If $f'(x) = x + \frac{1}{x}$, then $f(x)$ is

a) $x^2 + \log|x| + c$ b) $\frac{x^2}{2} + \log|x| + c$ c) $\frac{x}{2} + \log|x| + c$ d) $\frac{x}{2} - \log|x| + c$

2 The value of 'k' for which the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at x = 0 is

a) 0 b) -1 c) 1 d) 2

3 The interval of x in which $y = [x\,(x - 2)]^2$ is an increasing function is

a) $(-\infty, 0)$ b) $(0, 2)$ c) $(2, \infty)$ d) $(0, \infty)$

4 If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$ is equal to

a) $\frac{3}{2}$ b) $\frac{3}{4t}$ c) $\frac{3}{2t}$ d) $\frac{3}{4}$

5 The value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$ is

a) $\frac{13\pi}{7}$ b) $-\frac{13\pi}{7}$ c) $\frac{\pi}{7}$ d) $-\frac{\pi}{7}$

6 If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to

a) 5y b) 15y c) 25y d) 35y

7 The function $f(x) = \frac{4 - x^2}{4x - x^3}$

a) discontinuous at exactly one point
b) discontinuous at exactly three points
c) discontinuous at exactly two points
d) discontinuous at exactly four points

8 If $[x - 2 \quad 5+y]\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = \mathbf{0}$, then x + y is

a) 0 b) -1 c) -2 d) -3

9 Let $\sin^{-1}(2x) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$. Then the value of *'x'* is

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) 8

10 If A is a square matrix of order 2 and |A| = - 3, then the value of |5A| is

a) -3 b) -15 c) - 27 d) -75

11 Two positive numbers whose sum is 16 and the sum of whose cubes is minimum are

a) 8 & 8 b) 6 & 10 c) 4 & 12 d) 2 & 14

12 If A and B are square matrices of order 2, then det (A + B) = 0 is possible only when

a) det (A) = 0 or det (B) = 0
b) det (A) + det (B) = 0
c) det (A) = 0 and det (B) = 0
d) A + B = 0

13 $\int \frac{1}{sin^2x \,.\, cos^2x}$ dx is equal to

a) tan x + cot x + C b) -tan x + cot + C c) tan x - cot x + C d) -tan x - cot + C

14 The radius of the circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

a) 0.7 cm/sec b) π cm/sec c) 1.4π cm/sec d) 2π cm/sec

15 The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

a) 18 b) 27 c) 81 d) 512

16 $\int e^x$. sec x (1+tan x) dx equals

a) e^x cos x + C b) e^x sec x + C c) e^x sin x + C d) e^x tan x + C

17 The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (tan^5x + 1)$ dx

a) π b) 0 c) 1 d) 2

18 If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1}$ =_____

a) BA^{-1} b) $A^{-1}B^{-1}$ c) $A^{-1}B$ d) AB

Directions: In the following 2 questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is True

19 **Assertion (A):** If x is real, then the minimum value of $x^2 - 8x + 17$ is 1.
Reason (R): If f "(x) > 0 at a critical point, then the value of the function at the critical point will be the minimum value of the function.

a) b) c) d)

20 Consider the function *f*: R → R defined as $f(x) = x^3$
Assertion (A): *f*(x) is a one-one function.
Reason (R): *f*(x) is a one-one function if co-domain = range.
a) b) c) d)

SECTION – B

(Each Question Carries 2 Marks)

21 Find the real value of x for which the value of $\begin{vmatrix} 1 & -2 & 5 \\ 2 & x & -1 \\ 0 & 4 & 2x \end{vmatrix}$ is 86

22 (a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\,dx$

- OR -

(b) Evaluate $\int_1^3 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{4-x}}\,dx$

23 (a) Sketch the graph of $\sin^{-1} x$

- OR -

(b) Simplify $\sin^2\left(cos^{-1}\left(\frac{1}{4}\right)\right) + \cos^2\left(sin^{-1}\left(\frac{1}{3}\right)\right)$

24 Let the function *f*: R → R be defined by *f (x)* = cos x ∀ x ∈ R, show that f is neither one-one nor onto.

25 Find the value of '*a*' and '*b*' such that the function defined is a continuous function

$$f(x) = \begin{cases} 5, & if\ x \le 2 \\ ax + b, & if\ 2 < x < 10 \\ 21, & if\ x \ge 10 \end{cases}$$

SECTION – C

(Each Question Carries 3 Marks)

26 Find $\frac{dy}{dx}$ if x = a $\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and y = a sin θ

27 (a) Integrate the function $\tan^{-1}x$

- OR -

(b) Evaluate $\int_1^3 |x^3 - 2x|$ dx

28 The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres?

- OR -

A particle moves along the curve y = $\frac{2}{3}$ x^3 + 1. Find the coordinates of the points on the curve at which the y-coordinate is changing twice as fast as the x coordinate.

29 (a) Let *f* : N → R be a function defined as *f(x)* = $4x^2 + 12x + 15$, where R is the range of *f*. Show that the function *f(x)* is one-one.

- OR -

(b) Show that the function f: R → {x ∈ R : - 1< x < 1} defined by f(x) = $\frac{x}{1+|x|}$, x ∈ R is a one-one onto function

30 Express the matrix A = $\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

31 If y = 3 cos (log x) + 4 sin (log x), show that $x^2 y_2 + xy_1 + y = 0$

SECTION – D

(Each Question Carries 5 Marks)

32 (a) Show that the relation R defined on the set N × N by
(a, b) R (c, d) iff ad (b + c) = bc (a + d) is an equivalence relation.

- OR -

(b) Show that the relation R in the set A = {$x \in Z: 0 \le x \le 12$} given by
R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation.
Also find the equivalence class containing 1

33 Using matrix method, solve the system of equations

$x + 2y - 3z = -4$;
$2x + 3y + 2z = 14$;
$3x - 3y - 4z = -15$

34 Find $\frac{dy}{dx}$ if $x^y + y^x + x^x = a^b$

35 (a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

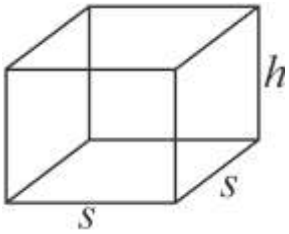
- OR -

(b) Evaluate $\int \frac{3x + 5}{x^3 - x^2 - x + 1}\, dx$

SECTION – E

(CASE STUDY - Each Question Carries 4 Marks)

36 An open tank is to be constructed using metal sheet with a square base and vertical sides so that it contains 500 cubic meters of water.



Using above information answer the following:

(i) Find the minimum surface area of the tank (2m)

(ii) Find the percentage increase in volume of the tank, if size of square base of tank become twice and height remains same. (2m)

37 Let $P(x) = -5x^2 + 125x + 37500$ is the total profit function of a bike manufacture company, where x is the production of the company.
Based on the above information, answer the following questions.

(i) What will be production of the company when the profit is ₹38250? (1m)

(ii) When the production is 2 unit, what will be profit of the company? (1m)

(iii) (a) Find the maximum profit of the company. (2m)

- OR -

(b) Find the intervals in which the profit is strictly increasing and decreasing.

38 A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹70.

(i) Convert the given above situation into a matrix equation of the form AX = B (1m)

(ii) Find |A| (1m)

(iii) (a) Find A^{-1} and cost of the pen 'C' variety (2m)

- OR -

(b) Find cost of each variety of pen (2m)

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